### Conclusions

Using previously developed theoretical techniques, stability boundaries for a buckled plate exposed to a static pressure differential have been computed and compared to newly available experimental data. Flutter boundaries for plates with both zero and complete in-plane edge restraint were calculated. The effect of the static pressure differential is stabilizing for both extremes of in-plane restraint, but is very much more so for the case of zero restraint than for complete restraint. The experimental data lie closest to the stability boundary for zero restraint. All of these results are consistent with similar calculations and experimental correlations made previously for pressure loaded plates free of applied in-plane loads.

The calculated flutter frequencies are not in as good agreement with the experimental results as are the flutter boundaries. The theoretical results are complicated somewhat by the existence of sustained oscillatory solutions in flow regimes where normally the panel would be stable with respect to "small" disturbances. These solutions are interpreted as responses to large disturbances of the panel structure. Such large disturbance oscillations differ substantially from the linear flutter frequency only for very small static pressure differentials (where, for a buckled plate, the flutter motion is strongly amplitude dependent).

### References


[1] Hess, R. W., "Experimental and Analytical Investigation of the Flutter of Flat Built-up Panels Under Streamwise In-Plane Load," TR-330, 1970, NASA.

[2] Dowell, E. H. and Voss, H. M., "Experimental and Theoretical Panel Flutter Studies in the Mach Number Range 1.0 to 5.0," ASD-TDR-63-449, June 1963, Aeronautical Systems Div., Wright-Patterson AFB, Ohio.

[3] Dixon, S. C. and Shore, C. P., "Effects of Differential Pressure, Thermal Stress and Buckling on Flutter of Flat Panels with Length-Width Ratio of 2," TN D-1949, 1963, NASA.

[4] Shideler, S. L., Dixon, S. C., and Shore, C. P., "Flutter at Mach 3 of Thermally Stressed Panels and Comparison with Theory for Panels with Edge Rotational Restraint," TN D-3498, 1966, NASA.

[5] Dowell, E. H., "Nonlinear Oscillations of a Fluttering Plate," *AIAA Journal*, Vol. 4, No. 7, July 1966, pp. 1267–1275.

[6] Dowell, E. H. and Ventres, C. S., "Nonlinear Flutter of Loaded Plates," AIAA Paper 68-286, Palm Springs, Calif., 1968.

[7] Ventres, C. S. and Dowell, E. H., "Comparison of Theory and Experiment for Nonlinear Flutter of Loaded Plates," *AIAA Journal*, Vol. 8, No. 11, Nov. 1970, pp. 2022–2030.

[8] Hess, R. W., Private communication, April 1970.


# Supersonic Membrane Flutter


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## 1. Introduction

Reference 1 has presented an interesting study of supersonic membrane flutter by considering the flutter of a two-dimensional plate in the presence of chordwise tensile in-plane stresses as the plate bending rigidity approaches zero. An asymptotic analysis using piston theory aerodynamics was developed based on the hypothesis of a boundary layer at the plate edge and several solutions were presented for analyses of various orders of approximation. These various solutions all showed that as the plate thickness approached zero, a simple flutter criterion was obtained viz.,

$$[2q/M\sigma_x][E^1/\sigma_x]^{1/2} = [\tfrac{2}{3}]^{3/2} \tag{1}$$

where $E^1 = E/12(1 - \nu^2)$ and the other symbols have their usual meaning.[1] The authors of Ref. 1 believed this to be a crude but conservative design criterion for the prevention of flutter of extremely thin two-dimensional plates at high Mach numbers.

Reference 2 has shown how the results of Ref. 1 may be generalized to include three-dimensional plates and other effects such as spanwise tensile in-plane stresses, structural damping, elastic foundation, orthotropicity, etc., based on the results of several other references, e.g., Ref. 3; and for Mach numbers from low subsonic through to high supersonic if the plate is of low aspect ratio.

The purpose of this Note is to show that the solution obtained in Ref. 1, as represented by Eq. (1), is a limiting result obtainable by the exact analysis of Ref. 4, which is a completely general study of supersonic flutter of flat rectangular orthotropic plates, and rather more general in its choice of boundary conditions than Ref. 3, which also contained exact analyses.

## 2. Analysis of Ref. 4

The governing differential equation for high supersonic speed flutter of a three-dimensional plate using two-dimensional static aerodynamics is given in Ref. 4 in terms of the chordwise deflection variable $X(x/a)$ as

$$X^{\mathrm{IV}}\left(\frac{x}{a}\right) + \pi^2\bar{A}X^{\mathrm{II}}\left(\frac{x}{a}\right) + \lambda X^{\mathrm{I}}\left(\frac{x}{a}\right) - \pi^4\bar{B}X\left(\frac{x}{a}\right) = 0 \tag{2}$$

where

$$\bar{A} = \left(\frac{a}{b}\right)^2\left[k_x + 2\left(\frac{D_{12}}{D_1}\right)\left(\frac{C_1}{\pi^2C_0}\right)\right] \tag{3}$$

$$\lambda = \frac{2qa^3}{\beta D_1}; \quad \beta = [M^2 - 1]^{1/2} \tag{4}$$

$$\bar{B} = \left(\frac{a}{b}\right)^4\left[\left(\frac{\omega}{\omega_0}\right)^2 - k_y\left(\frac{C_1}{\pi^2C_0}\right) - \left(\frac{D_2}{D_1}\right)\left(\frac{C_2}{\pi^4C_0}\right)\right] \tag{5}$$

and $k_x$, $k_y$ are nondimensionalized in-plane compressive resultants for chordwise and spanwise loadings $N_x$,$N_y$ respectively. $C_0$,$C_1$,$C_2$ are coefficients due to spanwise integrations of the spanwise modal deflection $Y(y/b)$ as indicated similarily in Ref. 2; $D_1$,$D_2$,$D_{12}$ are appropriate rigidities of the orthotropic plate. Equation (5) can be generalized further to include the effect of an elastic foundation stiffness $K$ as in Ref. 3, but this does not directly affect the critical flutter speed parameter $\lambda_{cr}$ as will now be discussed.

The general solution to Eq. (2) with due consideration of the boundary conditions at $x = 0$, $a$, has been found in Ref. 4, and for large negative values of the parameter $\bar{A}$ (corresponding to large tensile chordwise in-plane stresses) a simple algebraic solution for $\lambda_{cr}$ in terms of $\bar{A}$ is obtained as given below, which is applicable to simply supported or clamped boundary conditions on the spanwise edges $x = 0,a$;

$$\lambda_{cr} = [4\pi^3/3][10 - \bar{A}][(4 - \bar{A})/6]^{1/2} \tag{6}$$

The corresponding value of $\bar{B}_{cr}$, as given in Ref. 4 in terms of $\bar{A}$, defines the flutter frequency and it is seen therefore that the terms in $k_y$ and $K$ do not directly influence $\lambda_{cr}$.

Expanding the various terms in Eq. (6) for an isotropic plate one obtains

$$\lambda_{cr} = \frac{2qa^3}{\beta D} = \frac{4\pi^3}{3[6]^{1/2}}\left[10 - \frac{N_xa^2}{\pi^2D} + 2\,\frac{a^2}{b^2}\,\frac{C_1}{\pi^2C_0}\right] \times \left[4 - \frac{N_xa^2}{\pi^2D} + 2\,\frac{a^2}{b^2}\,\frac{C_1}{\pi^2C_0}\right]^{1/2} \tag{7}$$


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where $C_1/\pi^2 C_0 = -1$ for simply supported streamwise edges and $-1.236$ for streamwise edges, which are clamped.

### 3. Discussion

It follows from Eq. (7) for an infinite span panel ($a/b = 0$) and with $D \to 0$, that if $\bar{N}_x = -N_x$

$$\frac{2qa^3}{\beta} = \frac{4\pi^3}{3}\left[\frac{\bar{N}_x a^2}{\pi^2}\right]\left[\frac{\bar{N}_x a^2}{6\pi^2 D}\right]^{1/2} \tag{8}$$

i.e.,

$$[2q/\beta\sigma_x][E^1/\sigma_x]^{1/2} = [2/3]^{3/2} \tag{9}$$

or

$$[2q/\beta] = [2/3]^{3/2}[\bar{N}_x{}^3/D]^{1/2} \tag{10}$$

Clearly, when $M \gg 1$, $\beta \to M$ and Eq. (9) becomes identical to Eq. (1). Thus, Eq. (6), which is derived in Ref. 4 from an exact analysis for large negative values of $\bar{A}$, leads in the limiting case of $D \to 0$ to the result obtained by an alternative procedure in Ref. 1.

For an infinite span plate, but with $D > 0$, Eq. (7) can be adapted to become

$$\left[\frac{2q}{\beta\sigma_x}\right]\left[\frac{E^1}{\sigma_x}\right]^{1/2} = \alpha^2 = \left[\frac{2}{3}\right]^{3/2} \times [1 + 10\pi^2\epsilon^2][1 + 4\pi^2\epsilon^2]^{1/2} \tag{11}$$

where $\epsilon^2 = [E^1/\sigma_x][h/a]^2$ and Eq. (11) can be shown to give the form of Fig. 2 in Ref. 1. Conversely, if $N_x = 0$, Eq. (7) becomes

$$\lambda_{cr}\bar{\epsilon}^3 = \bar{\alpha}^2 = [\tfrac{2}{3}]^{3/2}[1 + 10\pi^2\bar{\epsilon}^2][1 + 4\pi^2\bar{\epsilon}^2]^{1/2} \tag{12}$$

where $\bar{\epsilon}^2 = [C_0/2C_1](b^2/a^2)$. Thus in the new notation $\bar{\alpha},\bar{\epsilon}$, Eq. (12) has the same form as Eq. (11) for different initial assumptions. This was also shown in Ref. 2.

Reference 4 has shown that the solution, Eq. (6), from which Eqs. (7–12) have been obtained, is a very good approximation to the results for simply supported spanwise edges for small negative values of $\bar{A}$ but is less good for clamped edges. The corresponding exact results for all negative $\bar{A}$ as given in Ref. 4 have been considered in terms of the terminology $\bar{\alpha},\bar{\epsilon}$, and it is worth noting that the curves thus obtained for simply supported and clamped spanwise edges, respectively, agree with those presented in Fig. 1 of Ref. 2.

### 4. Conclusions

The exact analyses of Ref. 4 have been shown to agree with those of other references—in particular those of Refs. 1–3 for three-dimensional plates at high supersonic Mach numbers. For membrane-type plates in which $D \to 0$ and in the presence of large tensile chordwise in-plane stresses,

$$[2q/\beta] = [2/3]^{3/2}[\bar{N}_x{}^3/D]^{1/2} \tag{10}$$

This appears preferable to that given in Ref. 1 and Eq. (9), in that the quantities $\bar{N}_x$ and $D$ are probably more easily defined than $\sigma_x$ and $E^1$. This is so because $\bar{N}_x$ is dependent only on the external loading and only $D$ is dependent on the material selected. For many membrane materials the determination of $E^1$ from the bending rigidity $D$ would require an estimation of the effective thickness $h$, as would $\sigma_x$ from $\bar{N}_x$. For a given material and flight condition, the required value of $\bar{N}_x$ to ensure stability is given from Eq. (10) by

$$\bar{N}_x = \tfrac{3}{2}D^{1/3}[2q/\beta]^{2/3} \tag{13}$$

### References

[1] Spriggs, J. H., Messiter, A. F., and Anderson, W. J., "Membrane Flutter Paradox—An Explanation by Singular Perturbation Methods," *AIAA Journal*, Vol. 7, No. 9, Sept. 1969, pp. 1704–1709.

[2] Dowell, E. H. and Ventres, C. S., "Flutter of Low Aspect Ratio Plates," *AIAA Journal*, Vol. 8, No. 6, June 1970, pp. 1161–1164.

[3] Dugundji, J., "Theoretical Considerations of Panel Flutter at High Supersonic Mach Numbers," *AIAA Journal*, Vol. 4, No. 7, July 1966, pp. 1257–1266.

[4] Erickson, L. L., "Supersonic Flutter of Flat Rectangular Orthotropic Panels Elastically Restrained against Edge Rotation," TN D-3500, Aug. 1966, NASA.

# Influence Coefficients for Pressurized Cylindrical Shells of Finite Length

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In this Note the influence coefficients for thin-walled cylindrical shells of finite length subjected to uniform internal pressure and axisymmetric edge loads are presented. Only the final results are given here; the details of the derivation may be found in Ref. 1. Both cases, in which the edge loads are symmetric or antisymmetric with respect to the central cross-sectional plane of the cylindrical shell are considered. The nonlinear coupling effect of the pressure loading and edge loads is taken into account.

The equations governing the stresses and deformations of a cylindrical shell under internal pressure and axisymmetric edge loads are those given by Nachbar.[2] In the derivation of the equations, the secondary state direct stresses due to the axisymmetric edge loads are assumed to be a small perturbation on the primary state direct stresses or membrane state of stress due to the uniform internal pressure.

With the usual notations (Figs. 1 and 2) and with the following definitions

$$\xi = \frac{x}{(ah)^{1/2}}[12(1-\nu^2)]^{1/4};\quad \lambda = \frac{l}{(ah)^{1/2}}[12(1-\nu^2)]^{1/4};$$
$$w = \frac{w_2}{a};\quad \rho = \frac{pa}{2h\sigma_c};\quad \sigma_c = \frac{E}{[3(1-\nu^2)]^{1/2}}\frac{h}{a};$$
$$H = \frac{2H^*}{h\sigma_c}\left[\frac{\sigma_c}{2E}\right]^{1/2};\quad M = \left(\frac{M^*a}{D}\right)\left(\frac{\sigma_c}{2E}\right);$$
$$D = Eh^3/[12(1-\nu^2)] \tag{1}$$

where $a$ is the radius, $2l$ is the length and $h$ is the thickness of the cylindrical shell; the governing differential equation can be written as

$$(d^4w/d\xi^4) - 2\rho(d^2w/d\xi^2) + w = 0 \tag{2}$$

where $\rho$ is the pressurization parameter defined in Eq. (1).

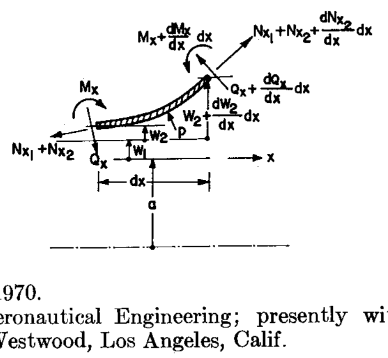

Fig. 1 Sign convention and forces on an element.

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